experimental conditions such as current, voltage, slit widths, etc., remain the same.

Results similar to the above have been obtained for secondary radiators composed of other chemical elements having atomic weights near that of molybdenum.

When chemical elements of low atomic weight such as carbon were used as secondary radiators we obtained ionization currents of about 0.1 or 0.2 millimeter per second representing the scattered tungsten $K\alpha$ radiation. In other words, the scattered radiation from the light chemical elements has a very much greater intensity than from the heavy, which agrees with previous findings.

The conclusions are that the scattered and tertiary radiation due to tungsten X-rays falling upon chemical elements of atomic weights near that of molybdenum are extremely weak as compared with the fluorescent radiation. They are too weak to be accurately studied with our present apparatus. The large currents in former experiments representing the scattered and tertiary radiation must have been due to some other cause than the radiation coming from the secondary radiator. The general stray radiation will not explain the tungsten peaks observed in the previous experiments and we do not know where they came from.

These very weak spectra will be investigated more thoroughly as soon as the new high tension storage battery has been completed and as soon as a water-cooled tungsten target tube which the General Electric Company kindly furnished us and which was accidentally burnt out has been repaired.

NOTE ON THE QUANTUM THEORY OF THE REFLECTION OF X-RAYS

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In a note published in these PROCEEDINGS for May, 1923, the writer proposed the fundamental principle of a theory of interference and diffraction phenomena based on quantum laws. According to this theory, when a quantum of radiation changes its direction in accordance with the observed phenomena of interference and diffraction, it transfers energy and momentum to the diffracting system. The transfer of momentum, according to the theory, takes place in quanta, the magnitudes of the quanta depending in a particular way upon the characteristics of the diffracting system.

This theory has been developed further by Breit,[1] A. H. Compton[2] and Epstein and Ehrenfest,[3] the latter having extended it to Fraunhofer diffraction in general.

In the original article, the theory was applied firstly to the reflection of an X-ray by a crystal in which the X-ray does not give up energy to an atom in the reflecting crystal, splitting off an electron. Then the case in which the X-ray does split off an electron and the atom subsequently emits some of the lines in its own spectra was discussed briefly and equations were deduced which indicate a rather strong reflection by the crystal of its own characteristic radiation. The object of this note is to examine the latter application of the theory in greater detail.

In order to apply the theory to an incident X-ray which splits off an electron and causes an atom in the crystal to radiate its fluorescent spectrum, an assumption had to be made, in addition to the fundamental laws of the theory. This additional assumption was that the total fluorescent radiation from an atom quantized its momentum with the crystal, the size of the quantum being determined, as in the general case, by the characteristics of the crystals. With this additional assumption the theory leads to the equations (7) of the original note. In general, they represent three-dimensional problems, but, for simplicity, we may consider an approximate, two-dimensional one. For reflection in a particular case the equations (7) reduce to

$$\begin{aligned} \Sigma \frac{1}{\lambda_1} (\cos\theta - \cos\theta_1) &= \frac{\tau_1}{d_1} \\ \Sigma \frac{1}{\lambda_1} (\sin\theta - \sin\theta_1) &= \frac{\tau_2}{d_2} \end{aligned} \qquad (1)$$

where θ is the glancing angle of incidence; θ_1, θ_2, etc., the angles of reflection of the various fluorescent rays, having the parameters that we call wave-lengths λ_1, λ_2, etc. d_1 and d_2 are two of the grating spaces in the crystal, and τ_1 and τ_2 are whole numbers, whose values determine the particular sets of planes in the crystal that do the reflecting and the orders of the reflections from them. Taking λ_1 to refer to the fluorescent ray of greatest energy emitted by the atom it appears that in any particular case, with τ_1 and τ_2 given, θ_1 can have only certain values whereas θ_2, θ_3, etc., are indeterminate. Assuming that all values of θ_2, θ_3, etc., are equally probable, that is, that the corresponding X-rays have the same probability of passing off from the crystal in all directions, we can calculate the possible values of θ_1 and also estimate the probability that θ_1 will have any one of its possible values.

Figure 1 represents graphically the relation between the angle of incidence θ and the possible values of θ_1 corresponding to it for the particular case $\tau_1 = 0$, that is, for reflections from let us say the 100 planes of the crystal. It will be seen that the points representing corresponding values of θ and θ_1 lie on a closed ellipse-shaped curve and that they are more or less concentrated about the ends of the curve. One end, A, of the curve lies

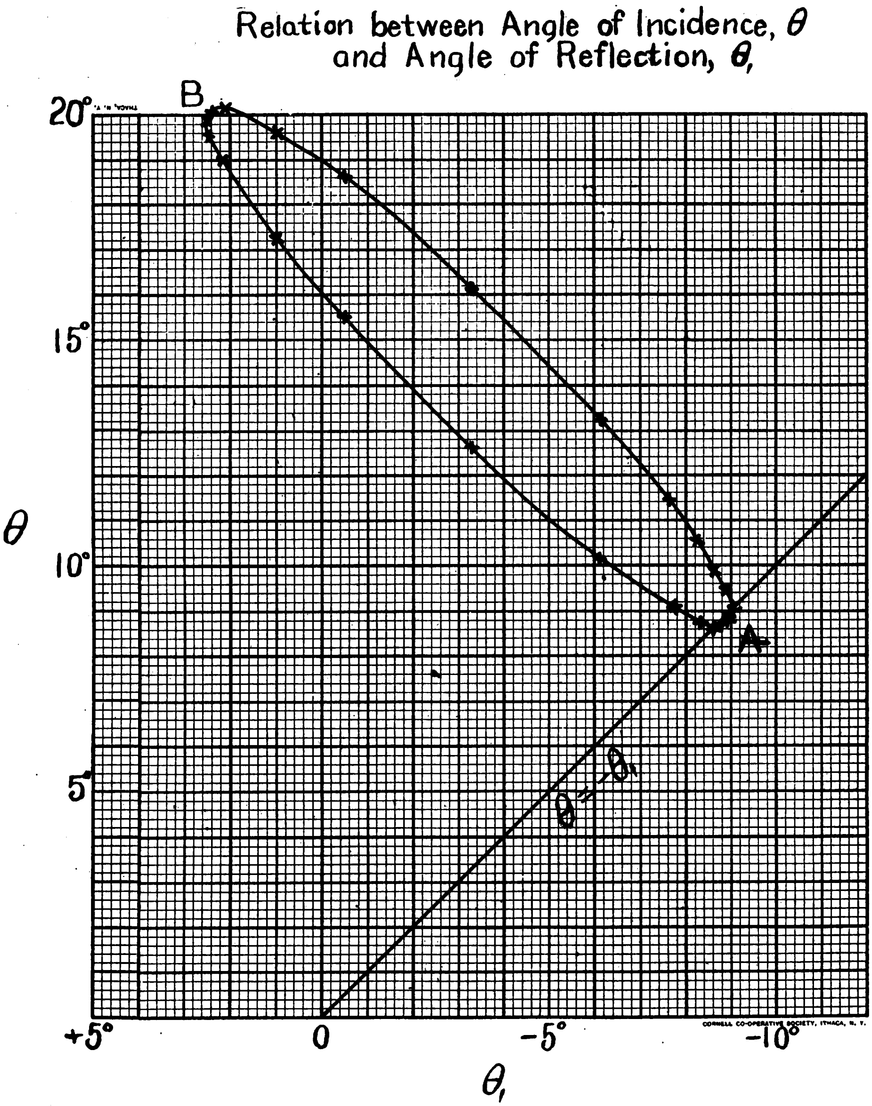

FIGURE 1

very close to the straight line representing $\theta = -\theta_1$, which corresponds to normal reflection of X-rays from the planes of the crystal under consideration. If an ionization spectrometer were set up with the angle of incidence, θ, equal to the angle of reflection, θ_1, it would register a maximum ionization current when this common angle had the proper value; for the current would be proportional to the number of points in the small rectangle $\Delta\theta \times \Delta\theta_1$, $\Delta\theta$ representing the angular width of the incident beam of rays and $\Delta\theta_1$, that of the ionization chamber's slit. The curve indicates, however, that there is another pair of values for θ and θ_1, namely, those corresponding to the other end of the curve at B, near which the points are concentrated. If the ionization chamber and crystal were so placed that the angle of incidence and the angle of reflection had the values represented by B the ionization chamber would register a maximum current of about the same magnitude as before and this would correspond to a kind of abnormal reflection of X-rays. This addition to the theory of the transfer of momentum in quanta leads to the conclusion that for every regular reflection, there must be an abnormal reflection of about equal intensity. The facts, however, do not warrant this conclusion. There are not as many strong abnormal reflections from a crystal as normal ones. In fact it seldom occurs that an abnormal reflection is as strong as the normal reflections. It follows, therefore, that this particular addition to the theory, namely, that only the total fluorescent radiation from an atom quantizes its momentum with the crystal and that the longer rays have equal probabilities of passing off in all directions, is not justified by experiment. If we look at the reflection of X-rays from the point of view of the quantum theory, we should assume that the law of the transfer of momentum in quanta applies to the individual fluorescent quanta emitted from an atom and not to the sum total of all the fluorescent rays only. This means that the reflection of characteristic rays should be weak, for only those rays that happen to pass off in the proper directions will strike the crystal planes at the correct angles for reflections. A large number of the rays whose direction angles lie within the limits $\theta \pm \delta\theta$, where θ is given by Bragg's law will be reflected. The magnitude of $\delta\theta$ and therefore the intensity of the reflected characteristic radiation depends partly on how nearly perfect the crystal is. These ideas have been discussed from the point of view of the wave-theory in previous papers.

[1] These PROCEEDINGS, July, 1923, p. 238.

[2] These PROCEEDINGS, Nov., 1923, p. 359.

[3] These PROCEEDINGS, April, 1924, p. 133.